A - LEVEL MATHEMATICS 2018/2019

STRUCTURE OF THE COURSE

- Your maths A-Level Maths course covers Pure Mathematics, Mechanics and Statistics.
- You will be examined at the end of the two-year course. The assessment will consist of three two-hour exams.

WORKLOAD & ORGANISATION

- You will have a heavy workload. Class time will be spent giving notes and explanations with questions and reinforcement set as homework.
- For <u>each</u> hour of taught time you should expect to spend an hour on maths work outside of lessons.
- All work will be done on paper and you must keep a well-organised file of notes and exercises.
- Each week you will be set a Core and Applied homework. In addition you will be expected to complete an independent exercise of your choice.

HOW TO BE A SUCCESSFUL A LEVEL STUDENT

The most successful students at A-level Mathematics:

- Recognise that A Level Mathematics is demanding and that they will have to work hard to understand the course fully (even if up until now they have always found maths easy)
- Participate in lessons, answering questions and asking questions about the ideas studied
- Keep a well organised folder of notes and use their notes to help them do the work set
- Seek help outside lessons before the deadline when they get stuck (despite having used their notes and had a go)
- Submit work on time that is complete
- Revise thoroughly with a focus on avoiding previous misconceptions / mistakes.

A-LEVEL MATHEMATICS - PREPARATORY WORK

This transition work covers the techniques that you need to be 100% confident on in order to be successful at A Level. This work consists of grade 4 - 8 GCSE algebraic manipulation. At GCSE such questions are often worth 3 marks but at A-level they will be worth 1 mark and will form only part of a more substantial question.

The problem solving questions are to show you the style of questions that are asked at A-level. These questions have been broken down into steps to make them more accessible but they remain challenging and you need to spend time working through them. You need to sure that your solutions are clear and well-structured as outlined in the accompanying example.

Read the instructions below carefully.

INSTRUCTIONS

All work should be completed on file paper not on the question sheets!

Essential Algebra

1 – Read through the examples on each section of the essential algebra before attempting the questions – each example is designed to help you avoid mistakes.

2 – Work through the questions showing your methods clearly – ie showing each step when rearranging a formula or solving an equation.

Problem Solving

1 – Read through the information about quality of written communication carefully.

2 – Work through the problems setting your work out clearly as demonstrated in the example.

We will not accept work that is set out poorly or is not of a good standard – if you attempt to hand in work like this you will be made to redo the entire section of work

NOTE

Your place on the A-level Mathematics course is dependent on completing the preparatory work to a high standard and being able to answer some similar questions under test conditions at the start of the course.

You should enjoy spending the time developing your algebra skills contained in this work as this is exactly the sort of thing you are committing yourself to by taking the A level course.

Essential Algebra

Section A: Expanding Brackets

The key to this section is knowing how to add, subtract and multiply negative numbers.

A1 $3(x^2 - y) - 2x(y - 4x)$ — Multiply out each bracket

$= 3x^2 - 3y - 2xy + 8x^2$ — Note that you are multiplying the second bracket by -2x not 2x and that: - 2x × -4x = + 8x^2

$= 11x^2 - 3y - 2xy$ — Collect the x^2 terms. No further simplification is possible.

A2 $(2x + 3y)(5x - 2y)$ — Multiply out to achieve 4 terms
Note that – x + = –

$= 10x^2 - 4xy + 15xy - 6y^2$ — Collect the term in xy carefully (noting that -4 + 15 = 11)

$= 10x^2 + 11xy - 6y^2$ — No further simplification is possible.

A3 $(x + 3y)^2$ — $(x + 3y)^2$ is **not** $x^2 + 3y^2$ nor is it $x^2 + 9y^2$!!

$= (x + 3y)(x + 3y)$ — Squaring means multiply by itself.

$= x^2 + 3xy + 3xy + 9y^2$ — Multiply out to achieve 4 terms

$= x^2 + 6xy + 9y^2$ — Collect the xy terms to simplify.

SECTION A QUESTIONS

Expand and simplify

1) $3(x – 2y) – 2x(y – 4)$
2) $(2x – 1)(3x + 2)$
3) $(3x – 5)(3x + 5)$
4) $(x – 5y)^2$
5) $5 – 3(x – 2)$ (hint BIDMAS)
6) $2(x + 1)^2$ (hint BIDMAS)
7) $(2x – 3y)(4x – 5y)$

Section B: Factorising Expressions

Factorising into 1 bracket (common factors)

B1 $12x^3y - 18x^2y^3$

- State the question. Look to factorise completely.

$= 6x^2y(\quad)$

- The highest number that each term divides by is 6.
- The highest power of x in each term is x^2.
- The highest power of y in each term is y.
- These are the common factors to be pulled outside of the bracket.

$= 6x^2y(2x - 3y^2)$

- Now divide each term by the common factors to get the contents of the bracket.
- The terms in the bracket have no common factors so you've factorised completely.
- Check by expanding that you've factorised correctly.

Factorising Quadratics (where the coefficient of $x^2 = 1$)

Standard 3 term quadratic…

B2 $x^2 + x - 12$

- It's a 3 term quadratic and it only has x^2 so if it will factorise it will go into 2 brackets (x)(x).

$(x + \quad)(x - \quad)$

- The –12 at the end means the numbers in the bracket must multiply to make -12. To get a negative by multiplying the signs must be different.

$(x + 4)(x - 3)$

- You need to try different combinations (-12 & 1, 12 & -1, -6 & 2, 6 & -2, -4 & 3, 4 & -3) and mentally multiply out until you get + 1x as your middle term.
- There is only one possible combination that works.

Only 2 terms – missing constant term…

B3 $x^2 + 5x$

- It's a 2 term quadratic with a missing constant term so if it will factorise into one bracket by common factor.

$x(x + 5)$

- The only common factor is x. There is only one possible combination that works.

Only 2 terms – missing x term…

B4 $x^2 + 5$

- This expression will not factorise. The only quadratics of this sort that will are the difference of two squares sort – see next example.

B5 $x^2 - 4$

$(x + \quad)(x - \quad)$

$(x + 2)(x - 2)$

- $x^2 - 4$ is a special kind of quadratic called the difference of two squares (dots!) because it is one square number subtract another square number.
- Dots factorises into (√1st term + √2nd term) (√1st term + √2nd term)
- Check by expansion that $(x + 2)(x - 2)$ gives $(x^2 + 0x - 4)$.

Factorising Quadratics (where the coefficient of $x^2 \neq 1$)

Look for common factors first…

B6

$2x^2 - 8x + 6$	○	It is a standard 3 term quadratic so will factorise into 2 brackets.
$= 2(x^2 - 4x + 3)$	○	There is a common factor of 2. Pull this out of each term to give an easier quadratic to factorise.
$= 2(x \quad)(x \quad)$	○	The + 3 at the end shows the signs in the brackets must be the same.
$= 2(x - \quad)(x - \quad)$	○	The – 4x is negative so the signs in the brackets must both be –.
$= 2(x - 3)(x - 1)$	○	This means the only possible values are –3 & -1.
	○	Check by expansion that $(x - 3)(x - 1)$ gives $(x^2 - 4x + 3)$.

B7

$9x^2 - 81$	○	State the question. There is a common factor of 9. Pull this out of each term to give an easier quadratic to factorise.
$= 9(x^2 - 9)$		
$= 9(x + \quad)(x - \quad)$	○	$x^2 - 9$ is a difference of two squares (dots!) because it is one square number subtract another square number.
$= 9(x + 3)(x - 3)$	○	Dots factorises into (√1st term + √2nd term) (√1st term + √2nd term)
	○	Check by expansion that $(x + 3)(x - 3)$ gives $(x^2 + 0x - 9)$.

Standard 3 term quadratic…

B8

$4x^2 + 8x - 5 = 0$	○	State the question. There are no common factors.
	○	It's a quadratic so if it will factorise it will go into 2 brackets but as it is $4x^2$ then it could be either $(4x \quad)(x \quad)$ or $(2x \quad)(2x \quad)$.
$(\_x + \quad)(\_x - \quad) = 0$	○	The –5 at the end means the signs in the bracket are different and the end nos are either 5 & -1 or –5 & 1.
	○	You need to try different combinations and mentally multiply out until you get + 8x as your middle term.
$(2x + 5)(2x - 1) = 0$	○	There is only one possible combination that works.

<u>SECTION B QUESTIONS</u>

Factorise <u>completely</u>

1) $6pq - 15p^2$

2) $x^2 - 36x$

3) $x^2 - 36$

4) $x^2 + 5x + 6$

5) $x^2 - 5x - 6$

6) $2x^2 - 14x + 12$

7) $6x^2 - 6$

8) $6x^2 + 7x + 2$

9) $4x^2 - 7x + 3$

10) $14x^2 - 3x - 2$

Section C: Solving Equations

Solving Quadratics by Factorising

C1 $4x^2 - 17x - 15 = 0$

- There are no common factors.
- It's a quadratic so if it will factorise it will go into 2 brackets but as it is $4x^2$ then it could be either (4x   )(x   ) or (2x   )(2x   ).

(__x +)(__x –) = 0

- The –15 at the end means the signs in the bracket are different and the end nos are either 15 & -1, -15 & 1, -5 & 3 or 5 & -3.
- You need to try different combinations and mentally multiply out until you get -17x as your middle term.

$(4x + 3)(x - 5) = 0$

- There is only one possible combination that works.

You must now solve the equation (ie find x). You have two things multiplied together that equal 0. This means that one or both of them must equal 0.

Either $(4x + 3) = 0$ which means $4x = -3$ so $x = -{}^{3}/_{4}$

Or $(x - 5) = 0$ $x = 5$

Solving Linear Simultaneous Equations

C2 Solve simultaneously $2x - 5y = 16$ (i)
$3x - 2y = 13$ (ii)

- To eliminate one variable you need the same amount of one of the variables in each equation. Multiply both sides of the first equation by 3 and both sides of the second equation by 2 to give 6x in each equation.

$6x - 15y = 48$ 3 × (i)
$6x - 4y = 26$ 2 × (ii)

- You now eliminate one variable by either adding or subtracting the equations. In this case you do 3(i) – 2(ii) Note that $-15y - (-4y)$ which is $-15y + 4y = 9y$ Now divide the equation through by -11 to find y:

$0x - 11y = 22$ 3(i) – 2(ii)
$y = 22 \div -11 = -2$

- Substitute this value of y back into the original equations to find x. Both equations should give the same x value!

in (i):	in (ii):
$2x - 5(-2) = 16$	$3x - 2(-2) = 13$
$2x + 10 = 16$	$3x + 4 = 13$
$2x = 6$	$3x = 9$
$x = 3$	$x = 3$

- The solution to the simultaneous eqns is $x = 3$, $y = -2$

SECTION C QUESTIONS

Factorise and then solve these quadratic equations.

1) $x^2 + 13x + 40 = 0$
2) $t^2 - 5t + 6 = 0$
3) $p^2 - 4p = 0$
4) $g^2 - 100 = 0$
5) $4x^2 - 4x - 3 = 0$
6) $6y^2 - 5y - 6 = 0$
7) $8x^2 - 2x - 1 = 0$
8) $2x^2 + 5x + 3 = 0$

Solve these simultaneous equations

9) $4x - 3y = 19$ (i)
$3x - 2y = 14$ (ii)

10) $2x + 4y = 2$ (i)
$3x - 5y = -19$ (ii)

Section D – Rearranging Formulae (I)

Formulae are constructed using the order of operations, BIDMAS…

Brackets	1st
I ndices (powers)	2nd
Division	}3rd
Multiplication	}
Addition	}4th
Subtraction	}

If you rearrange a formula you are 'unpicking it' so you must follow BIDMAS in reverse and do the opposite operation to both sides.

D1 Make p the subject of this formula $y = \frac{4p+3}{7}$

This is the same as y = (4p + 3) ÷ 7 because all of the 4p + 3 is in the numerator of the fraction.
Think about how it was constructed… p (×4) (+3) (÷7) = y.
To make p the subject you must follow these steps backwards and do the opposite.

Solution:

$$y = \frac{4p+3}{7}$$

- To undo the ÷7 you ×7 both sides $7y = 4p + 3$
- To undo the +3 you –3 both sides $7y - 3 = 4p$
- To undo the ×4 you ÷4 both sides $\frac{7y-3}{4} = p......hence....p = \frac{7y-3}{4}$

D2 Make p the subject of this formula $y = \frac{4p}{7} + 3$

Note this appears very similar to example E1 but is not the same because it was constructed in a different order.
This was constructed… p (×4) (÷7) (+3) = y.
To make p the subject you must follow these steps backwards and do the opposite to both sides.

Solution:

$$y = \frac{4p}{7} + 3$$

- To undo the +3 you –3 both sides $y - 3 = \frac{4p}{7}$
- To undo the ÷7 you ×7 both sides $7(y-3) = 4p$
- To undo the ×4 you ÷4 both sides $\frac{7(y-3)}{4} = p$

$$p = \frac{7(y-3)}{4} \quad \text{or} \quad p = \frac{7y-21}{4}$$

D3 Make p the subject of this formula $y = \frac{4p}{7} + 3$

This is exactly the same question as example E2. However there is another method to rearrange it…

You can eliminate the denominator of 7 by **multiplying through** by 7 – that mean multiplying the whole of each side by 7. In effect you multiply each term by 7

Solution:

$$y = \frac{4p}{7} + 3$$

- Multiply **every term** by 7 $(y \times 7) = (\frac{4p}{7} \times 7) + (3 \times 7)$
- Simplify (note: denominator is now gone) $7y = 4p + 21$
- To undo the ×4 you ÷4 both sides $7y - 21 = 4p......hence....p = \frac{7y-21}{4} or...p = \frac{7(y-3)}{4}$

Both methods (from E2 and E3) are equally good. The method of multiplying through is very useful but you must remember to multiply every term.

D4 Make g the subject of this formula $m = f - \frac{6g}{5}$

- The problem with this formula is that the subject that you want, g,is negative.
- The easiest way to solve this question without making any errors with minus signs is to add the whole of the term involving g to both sides.
  This now gives a formula with a positive term in g.
- Now follow the previous method - look at how the formula was made…g (×6) (÷5) (+m) = f
- Reverse the order of the steps and do the opposite to both sides of the formula.

$$m + \frac{6g}{5} = f - \frac{6g}{5} + \frac{6g}{5}$$

$$m + \frac{6g}{5} = f$$

$$\frac{6g}{5} = f - m$$

$$6g = 5(f - m)$$

$$g = \frac{5(f-m)}{6}$$

(Note: the order of construction as g (÷5) (×6) (+m) = f would also be correct)

SECTION D QUESTIONS

Make the letter in brackets the subject of the formula.

1. p = aq – r (q)
2. p = a(q – r) (q)
3. $y = \frac{2p}{3} + r$ (p)
4. $y = \frac{2p+r}{3}$ (p)
5. $y = r - \frac{qp}{x}$ (p)
6. $a^2 = b^2 + c^2$ (b)

Section E – Rearranging Formulae (II)

E1 Make g the subject of this formula $4f + h = \frac{3}{g}$

- The problem with this formula is that the subject that you want, g, is in the denominator of a fraction.
- The way to solve this problem is to multiply both sides by g. You must put brackets around the 4f + h to indicate that both terms are multiplied by g.This now gives a formula with g no longer in the denominator.
- Now you have g multiplied by a bracket equals 3 so divide both sides by the content of the bracket to get g =…

$$4f + h = \frac{3}{g}$$
$$g \times (4f + h) = \frac{3}{g} \times g$$
$$g(4f + h) = 3$$
$$g = \frac{3}{(4f + h)}$$
$$hence....g = \frac{3}{4f + h}$$

E2 Make x the subject of this formula $y - mx = cx + 4$

- The problem with this formula is that the subject you want, x, appears twice in the formula.
- The way to solve this problem is to collect all the terms With x in on one side and all the terms without x in them on the other side.
- To do that add mx to both sides and –4 to both sides
- You can now pull x out as a common factor by factorising
- Now you have x multiplied by a bracket equals y - 4 so Divide both sides by the content of the bracket to get x =…

$$y - mx = cx + 4$$
$$y = cx + 4 + mx$$
$$y - 4 = cx + mx$$
$$y - 4 = x(c + m)$$
$$\frac{(y-4)}{(c+m)} = x$$
$$hence...$$
$$x = \frac{y-4}{c+m}$$

SECTION E QUESTIONS

Make the letter in brackets the subject of the formula.

1. $y = \frac{2p}{q} + r$ (q)
2. ax + b = cx + d (x)
3. $p = \frac{5t - r}{u}$ (u)
4. $p = \frac{5t - u}{u}$ (u)
5. p(q + r) = 2(q – p) (q)
6. p(r – q)= 2 + 3(q – r) (r)
7. $y = r - \frac{qp}{x}$ (x)

Section F – Algebraic Fractions

F1: Simplify:

$$\frac{x+1}{3}-\frac{x-3}{2}$$

- The first thing we need when adding or subtracting is a common denominator:

$$\frac{2(x+1)}{6}-\frac{3(x-3)}{6}$$

- Now express as a single fraction and then simplify the numerator – be careful with signs!!

$$=\frac{2(x+1)-3(x-3)}{6}$$

$$=\frac{2x+2-3x+9}{6}$$

$$=\frac{-x+11}{6}$$

F2: Solve this equation:

$$\frac{3}{x-1}-\frac{2}{x+1}=1$$

- First write as a single fraction.

$$\frac{3(x+1)-2(x-1)}{(x-1)(x+1)}=1$$

- Then multiply both sides by the denominator to get rid of the fraction.

$$3(x+1)-2(x-1)=(x-1)(x+1)$$

- Expand the brackets and simplify – sometimes you may get a quadratic!!

$$3x+3-2x+2=x^2-1$$

- As this is a quadratic, rearrange so it is equal to zero.

$$x^2-x-6=0$$

- Factorise and solve:

$$(x-3)(x+2)=0$$

$$x=3\ or-2$$

Section F Questions:

Simplify:

1. $\frac{x+1}{2}-\frac{x-1}{3}$
2. $\frac{2x-1}{5}+\frac{1-x}{7}$
3. $\frac{6p}{5}-\frac{4p-3q}{3}$
4. $\frac{x}{2}-\frac{1-x}{3}+\frac{2-x}{4}$
5. $\frac{1}{x^2}-\frac{1}{x(x+1)}$

Solve:

6. $\frac{x+1}{2}+\frac{x+2}{5}=3$
7. $\frac{x+2}{5}+\frac{x+1}{7}=3$
8. $\frac{4x+1}{3}-\frac{x+2}{4}=2$
9. $\frac{2}{x+1}+\frac{5}{x+2}=3$
10. $\frac{4}{x-2}+\frac{7}{x+1}=3$

Section G – Rules of Indices

- $x^a \times x^b = x^{a+b}$ E.g. $4x^3 \times 3x^5 = 12x^8$
- $x^a \div x^b = x^{a-b}$ E.g. $\frac{3x^2 \times 6x^4}{9x^2} = \frac{18x^6}{9x^2} = 2x^4$
- $(x^a)^b = x^{ab}$ E.g. $(4x^7)^3 = 64x^{21}$
- $x^{-a} = \frac{1}{x^a}$ E.g. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$
- $x^{\frac{1}{n}} = \sqrt[n]{x}$ E.g. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
- $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ E.g. $27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = 3^4 = 81$
- $x^0 = 1$

G1 Write $\frac{1}{3x}$ as a single power of x

- Remember $\frac{1}{3x}$ is the same as $\frac{1}{3} \times \frac{1}{x}$
- Now, what is $\frac{1}{x}$ as a power of x?

$$\frac{1}{3x} = \frac{1}{3} \times \frac{1}{x} = \frac{1}{3} \times x^{-1} = \frac{1}{3}x^{-1}$$

G2 Write $\frac{4}{\sqrt{x}}$ as a single power of x

- Write $\sqrt{x}$ as a power of x
- Now how can you rewrite this so $x^{\frac{1}{2}}$ is not in the denominator

$$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}}$$

Section G Questions

Simplify the following:

1. $4(a^3)^5 \div 2a^7$
2. $8(x^5)^2 \div 2x^6$
3. $\frac{15a^2b^7}{5ab^{-2}}$
4. $\frac{15(a^2b)^3}{(3ab)^2}$
5. $\frac{3a^2bc^3 \times 4a^2bc}{2a^2b^4c^3}$

Evaluate:

6. $2^{-1} + 4^{-1}$
7. $\left(\frac{3}{2}\right)^2$
8. $\left(\frac{3}{2}\right)^{-2}$
9. $\left(\left(\frac{2}{5}\right)^{-1}\right)^3$
10. $\left(\frac{3}{4}\right)^{-3} \div 6^{-2}$

Write the following as a single power of *x*.

11 $\frac{1}{x}$

12 $\sqrt[5]{x^2}$

13 $\frac{1}{x^7}$

14 $\frac{1}{\sqrt[3]{x}}$

15 $\sqrt[4]{x}$

16 $\frac{1}{\sqrt[3]{x^2}}$

Section H: The quadratic formula

You will be required to know the formula used to solve quadratic equations that cannot factorise.

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

H1 Solve $5x^2 - 11x - 4 = 0$, correct to two decimal places.

- Take the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Put a = 5, b = - 11 and c = -4: $x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \times 5 \times -4}}{2 \times 5}$
- Simplify as far as possible $x = \frac{11 \pm \sqrt{121 + 80}}{10} = \frac{11 \pm \sqrt{201}}{10}$
- Type into your calculator and round: $x = 2.52 \ or - 0.32$

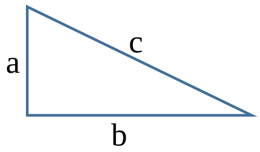
Section H Questions:

Solve the following equations using the quadratic formula.
Give your answers to two decimal places.

1. $2x^2 + x - 8 = 0$
2. $x^2 - x - 10 = 0$
3. $7x^2 + 12x + 2 = 0$
4. $4x^2 + 9x + 3 = 0$
5. $3x^2 - 7x + 1 = 0$
6. $4x^2 - 9x + 4 = 0$

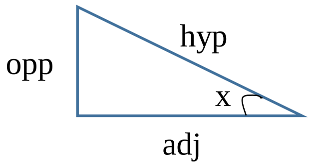
<u>**Section I: Pythagoras and Trigonometry**</u>

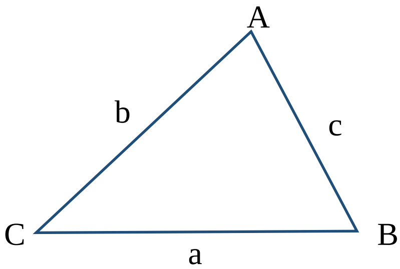
Pythagoras' Theorem: $a^2 + b^2 = c^2$



Trigonometry:

- $\sin x = \frac{opp}{hyp}$
- $\cos x = \frac{adj}{hyp}$
- $\tan x = \frac{opp}{adj}$





Cosine Rule:
You can use the cosine rule to find the length of a side when two sides and the included angle are given.

- $a^2 = b^2 + c^2 - 2bc \cos A$

Alternatively, you can use cosine rule to find an unknown angle if the lengths of all three sides are given.

- $\cos A = \frac{b^2+c^2-a^2}{2bc}$

Sine Rule:
You can use sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

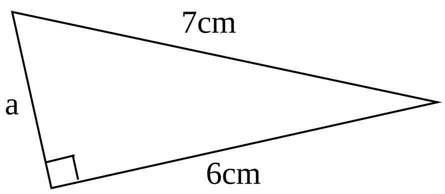
Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.

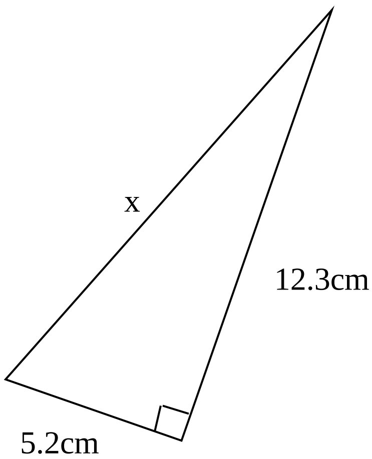
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Area of a Triangle:
The area of a triangle is $\frac{1}{2}ab \sin C$

Section I Questions

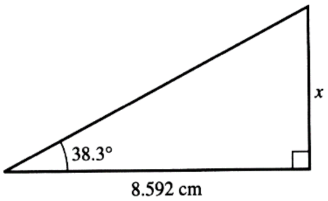
1. Find the missing side on the following triangles:



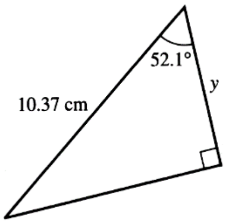


2. Find the value of the missing side or angle correct to 3 significant figures.

a



b



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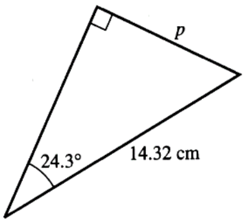

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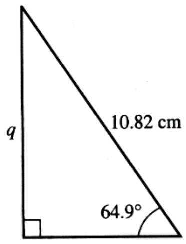

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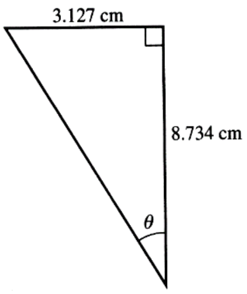

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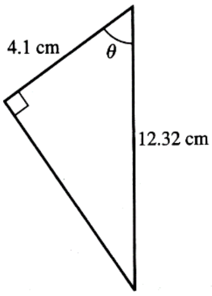
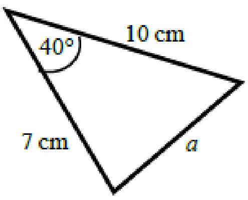


3. Work out the unknown side or angle labelled with a letter in each triangle.
Give your answers correct to 3 significant figures.

a



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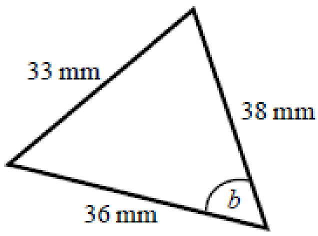

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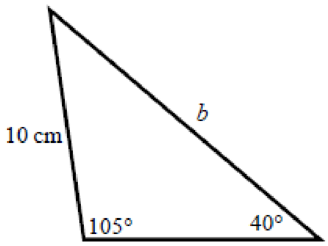

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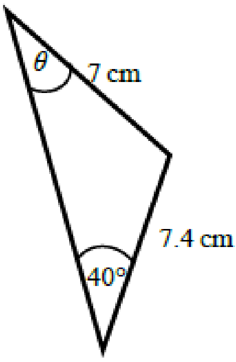
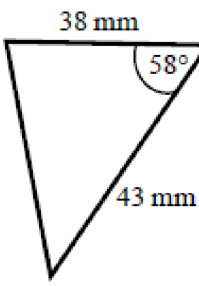


4. Work out the area if the triangle.



Problem Solving

This set of problems is designed to show you the kind of work that A level starts with. If you enjoy doing these questions, even when you find them difficult, then you are likely to enjoy the A level course. In doing this work you should concentrate as much on communicating your answers clearly as you do on solving the problems.

Look critically at the answers below which show two approaches to solving the same problem. Which do you think is better and why?

First Answer: $6 \times \frac{4}{3}\pi r^3 = 8\pi r^3$

$8\pi r^3 \div \pi r^2 = 8r$

$2\pi r^2 + 2\pi r(8r) = 18\pi r^2$

Second answer: Vol of a sphere = $\frac{4}{3}\pi r^3$

Vol of 6 spheres = $6 \times \frac{4}{3}\pi r^3 = 8\pi r^3$

Vol of 6 spheres = vol of cylinder with same radius: $8\pi r^3 = \pi r^2 h$

Finding h in terms of r: $h = 8\pi r^3 \div \pi r^2 = 8r$

Surface Area of cylinder = $2\pi r^2 + 2\pi rh$

= $2\pi r^2 + 2\pi r(8r)$

= $2\pi r^2 + 16\pi r^2 = 18\pi r^2$

IMPORTANT!

Quality of written communication [QWC] isn't just about setting your work out well. It enables you to structure your thinking and helps you to work through complicated problems as it makes you explain your method. It also makes it easy for other people to follow your work which makes it more likely that they will be able to help you if you get stuck!

Features of quality of written communication:

(i) You can understand what the question was from looking only at the solution
(ii) Working is clearly labelled… eg: Vol of sphere =
(iii) Formulae are stated before they are used
(iv) The algebraic steps 'flow' ie: They work down the page with each step equivalent to the one before and there is no misuse of the equals sign…

eg: GOOD: A = ½ bh
= ½ × 3 × 4
= 6 [cm^2]

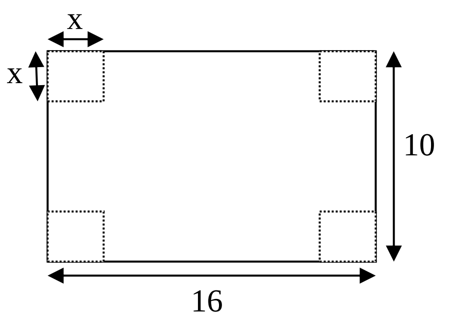
BAD: A = ½ bh
A = 3 × 4 = 12 ÷ 2 = 6 [cm^2]

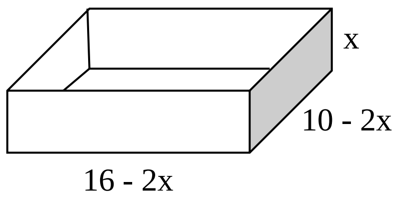
In the two answers at the start of this section the second answer has all the features of QWC and you should have been able to deduce that the original question was:
"6 spheres of radius, r, are melted down and reformed to make a cylinder also with radius r. Find the surface area of the cylinder in terms of r.

Problem Solving Questions

You need to attempt each question. These questions are designed to develop your problem solving skills and develop your resilience when tackling challenging problems. Some topics covered in the questions are new to you but you have been taught the skills required to solve the problems at GCSE. When we mark this transition work we are more interested in how you approach and tackle the problems than you getting the correct answer.
One mark will be awarded in each question for the quality of your written communication.

(1) An open topped tray is made from a rectangular piece of paper 16cm by 10cm by cutting out a square of side length x from each corner and folding.





a. Find an expression for the volume of the box giving your answer in the form $ax^3 + bx^2 + cx$ where a, b and c are integers. [3]
b. Show that the external surface area of the box is $160 - 4x^2$ [3]
c. Given that the external surface area is $96cm^2$, find the dimensions of the box. [3]

QWC [1]

(2) [You may find it useful to draw a sketch in this question – however you will not score any marks by solving the question using an exact graphical method]

Points A and B have coordinates (-1, 5) and (3, 7)

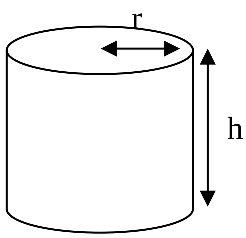
a. Show that the line that passes through A and B has a gradient of ½. [1]
b. M is the midpoint of AB. Find the coordinates of M. [1]
c. Line L is the perpendicular bisector of AB. This means that line L is perpendicular to the line through A and B and it also passes through the midpoint of AB.
Using the fact that the gradients of perpendicular lines multiply to make -1, find the gradient of line L. [1]
d. Using your gradient from(c) and the point M from (b), find the equation of line L. [3]

QWC [1]

(3) A cylinder has radius r and height h.

a. Write down in terms of r and h the formula for the volume of the cylinder. [1]
b. Show that the surface area of the cylinder is given by $2\pi r(r + h)$. [2]
c. Given that the surface area of the cylinder is four times its volume find an expression for the height of the cylinder in terms of the radius r. [3]

QWC [1]



(4) Show that the equation $6x + {}^{12}/_x = 17$ can be rearranged in the form $ax^2 + bx + c = 0$ where a, b and c are integers. Hence solve $6x + {}^{12}/_x = 17$ giving your answers as fractions. [3]

QWC [1]

(5) The equation of a circle with radius 5 units and its centre at the origin is $x^2 + y^2 = 25$.

a. On a single diagram draw $x^2 + y^2 = 25$ and the line $y = 5 – x$. [2]

b. By substituting $y = 5 – x$ into $x^2 + y^2 = 25$ show that you get the equation $2x^2 – 10x = 0$ [2]

c. Factorise $2x^2 – 10x$ <u>completely</u> [1]

d. Hence solve $2x^2 – 10x = 0$ [1]

e. How do the solutions to (d) relate to the diagram from (a)? [1]

QWC [1]

MECHANICS

(6) In formulae relating to motion: u = initial velocity, v = final velocity, a = acceleration, t = time and s = displacement.

Two formulae relating to motion of objects are: $s = ut + ½ at^2$ and $v = u + at$

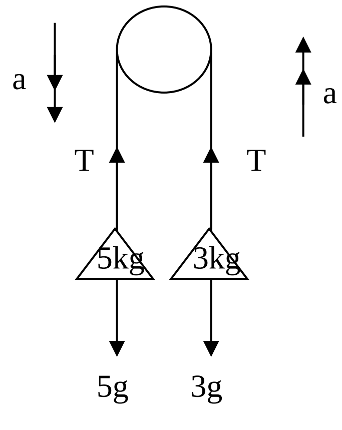
An object is travelling along a straight road at a velocity of 3 m/s when it starts to accelerate at 4 m/s^2. It accelerates until it has travelled 65 m.

a. Use $s = ut + ½ at^2$ to work out how long the object accelerated for. [4]

b. Use $v = u + at$ to work out the velocity of the object once it had stopped accelerating. [2]

QWC [1]

(7) The diagram shows two objects connected by a light inextensible string over a smooth pulley. Initially the weights are held at rest.



The **force** called tension in the string is marked 'T'.
The **force** called weight of each object is its mass [in kg] multiplied by gravity [called g].
When the objects are allowed to move they will accelerate in the directions shown at 'a' m/s^2

The force acts in the direction of motion. As the 5kg object will move downwards the force that acts on it is $5g – T$.

Newton's second law states that Force = Mass × Acceleration.
For the 5kg particle this gives the equation: $5g – T = 5a$.

a. Given that the 3kg object moves upward, use Newton's second law to give an equation for the 3kg particle. [2]

b. Solve the two equations simultaneously to show that $a = g/4$. [2]

c. Find T in terms of g, giving your answer as at fraction. [2]

QWC [1]

STATISTICS

(8) To find the mean of list of data you add all of the data values in the list and then divide by how many data values there are. In statistics the formula for the mean is $\bar{x} = \frac{\sum x}{n}$. $\bar{x}$ is the mean, Σ represents 'the sum of', x is a data value from the list and n is the number of values in the list.

The standard deviation is an alternative measure of spread to the range or interquartile range. The smaller the standard deviation, the more consistent the data is.

The standard deviation of a set of data is calculated as $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

a. Daniel records his scores in the weekly mental test for 6 weeks: 8, 5, 9, 11, 12, 15.
Find Daniel's mean score. [1]

b. Show that the standard deviation in Daniel's scores is 3.46 given to 3 significant figures. [3]

c. Paul also records his six test scores. Paul's mean score is 9.8 and his standard deviation is 1.5. Write two comments comparing the test performances of the two boys. [2]

QWC [1]

(9) Bag A and Bag B contain only Blue and Yellow counters
Bag A has 4 Blue counters and 1 Yellow counter.
Bag B also has five counters.

A counter is taken at random from Bag A and put into Bag B. Then a counter is taken from bag B and put into Bag A.
Bag A now contains only Blue counters.
The probability of this happening is $^{1}/_{10}$.

a. How many Yellow counters are in Bag B at the start? [4]

b. Before the counters were swapped if one counter was taken from each bag what would have been the probability of getting one of each colour? [2]

QWC [1]

TOTAL = 64 MARKS